The Value of Liquidity Sacrifice Options

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Financial Theory and Illiquidity

Approach to Transaction Costs	Examples
Ignored: assume zero	CAPM, Black-Scholes, first and second fundamental theorems of asset pricing, equivalent martingale measures. etc.
Fixed or proportional costs; trades driven to maintain asset proportions or option hedge.	Leland (1995), Davis and Norman (1990), Davis, Panas & Z (1993). Movable boundary problem for portfolio rebalancing. Convex set of EMMs (Ortu, 2000)
Multiple asset classes. Forced sales generated from capital structure. Assume the most liquid assets sacrificed first.	This presentation.
Endogenous illiquidity models.	Kim & Verrecchia (1994). Illiquidity is a consequence of information asymmetries. Bid-offer spreads compensate market-makers for adverse selection.

>150 Years of Illiquid Assets

They [life assurers] engage to pay a fixed sum of money at periods generally long distant from the time when the contracts are entered into … the probable amount of demands on their resources can be calculated from time to time within not very wide limits. Life assurance Societies, unlike banks and commercial enterprises generally, are not exposed to sudden or unusual demands on their resources in times of panic and financial difficulty. …
The much larger proportion [of life office assets] may safely be invested in securities that are not readily convertible; and it is desirable … that it should be so invested, because such securities, being unsuited for private individuals and trustees, command a higher rate of interest in consequence.

Arthur Hutcheson Bailey, Journal of the Institute of Actuaries, 1862.

Illiquidity Shocks in Insurance

Policy Drivers

Catastrophe insurance payout
Loss of confidence / publicity
No MVA dates on UWP
Embedded option moneyness
New product launch/churn
Optional additional premium

Market Drivers

Delta hedging
Other guarantee hedging
Hedge rollover
Group fungibility constraints
Derivative physical delivery
Collateral posting (derivatives)

Credit Drivers

Downgrades effect on

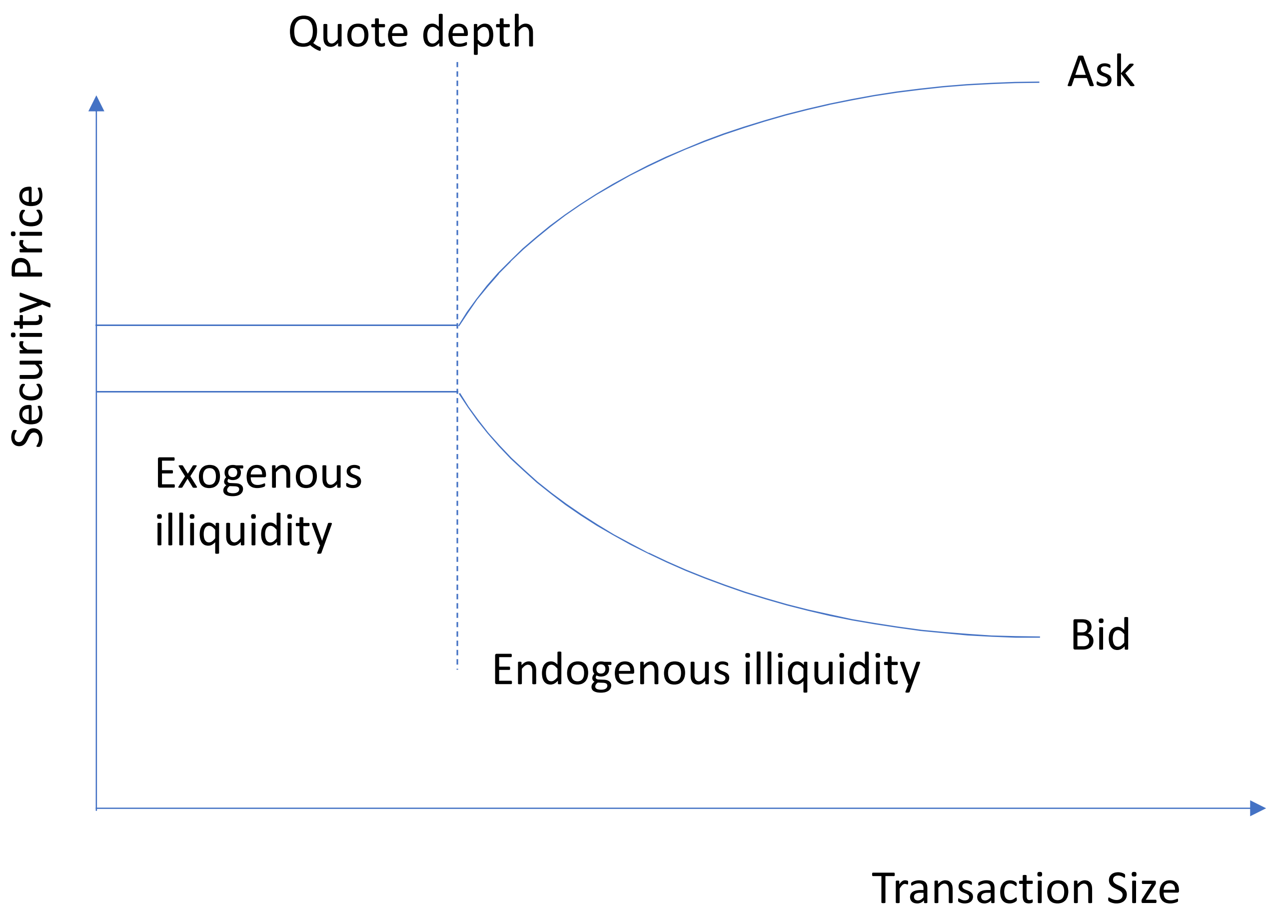
- Investment risk appetite
- Collateral quality
- Tracking an index

Accelerated settlement
Collateral liquidation

Financing Drivers

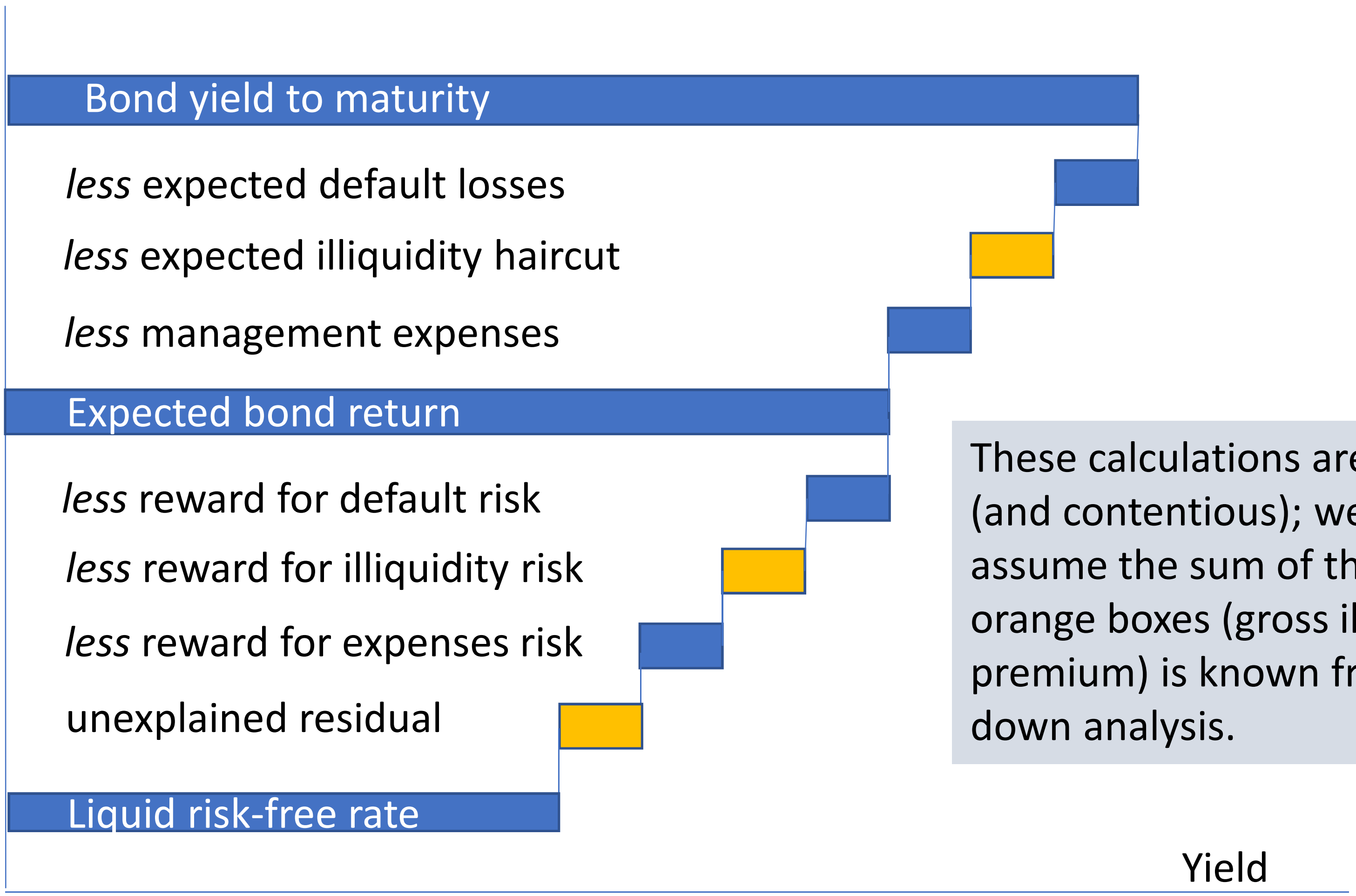
Debt coupons / principal
Merger / acquisition finance
Collateral payments on securitisation

Haircut Models



Source: Bangla, Diebold, Schuermann & Stroughair (2008)

Decomposing Bond Spreads



These calculations are tricky (and contentious); we shall assume the sum of the orange boxes (gross illiquidity premium) is known from top-down analysis.

Portfolio Optimisation Example

- Illustrative simplified market assumptions

Illiquidity rank	Asset Class	Haircut (normal sale)	Haircut h_j (forced sale)	Gross Illiquidity premium α_j
1	Cash	0%	0%	0%
2	Corporate bonds	0%	10%	0.5%
3	Private bonds	0%	40%	0.75%

- Normal sales at fair value; haircut is a discount in achieved price relative to fair value on forced sale.
- Illiquidity premium an increasing, concave, function of the haircut.
- Ignore interest rate, default and other market risks.

Investor Characteristics

- Return Period *r* as a Function of Shock Severity, measured as proportion *s* of portfolio (by fair value) subject to forced sale.

Liquidity Shock	≥1%	≥10%	≥*s*
Hedge fund	1 year	2 years	
Life Fund	5 years	10 years	
Annuity Fund	20 years	50 years	
General case return period	$r(0.01)$	$r(0.1)$	$r(s)$

- This is sometimes called an ‘operational risk’ model because it is mathematically similar to some OR models under the Basel Capital Accord.
- Related to the poorly defined concept of ‘liability illiquidity’.
- We have made the simplifying assumption of being able to choose which assets to sacrifice up to the shock size *s*. This is the liquidity sacrifice option.

Optimisation: Hedge Fund

- For the hedge fund:
  - Linear extrapolation of shock frequencies:

Shock *s*	**1%**	**10%**	**100%**
Return period *r*(*s*)	1 year	2 years	12 years

  - We are forced to realise the entire fund once every 12 years (in addition to more frequent smaller shocks)
  - Illiquidity premium net of expected costs is at best the gross illiquidity premium minus one twelfth of the haircut.
  - But the illiquidity premium is less than 1/12 of the haircut, for both of the illiquid asset classes.
  - So neither of the illiquid assets are attractive; best to hold 100% in cash.

Objective Function

- Idea: net illiquidity premium β = gross illiquidity premium α minus expected illiquidity haircut.

$$\beta = \alpha - \frac{h}{r}$$

- Suppose portfolio weight w_j in liquidity classes 1+2+..+j, so weight in class j is w_j-w_{j-1}≥0. Set w_0 = 0 and w_{jmax}=1.
- Maximisation objective

$$\beta = \sum (w_j - w_{j-1})\alpha_j - \sum \left\{ h_j \int_{w_{j-1}}^{w_j} \frac{ds}{r(s)} \right\}$$

- This is a straightforward convex optimisation problem
  - For interior solutions, equate marginal net IP

$$\alpha_j - \frac{h_j}{r(w_j)} = \alpha_{j+1} - \frac{h_{j+1}}{r(w_j)}$$

  - Care at corner solutions where some w_j=w_{j-1}.

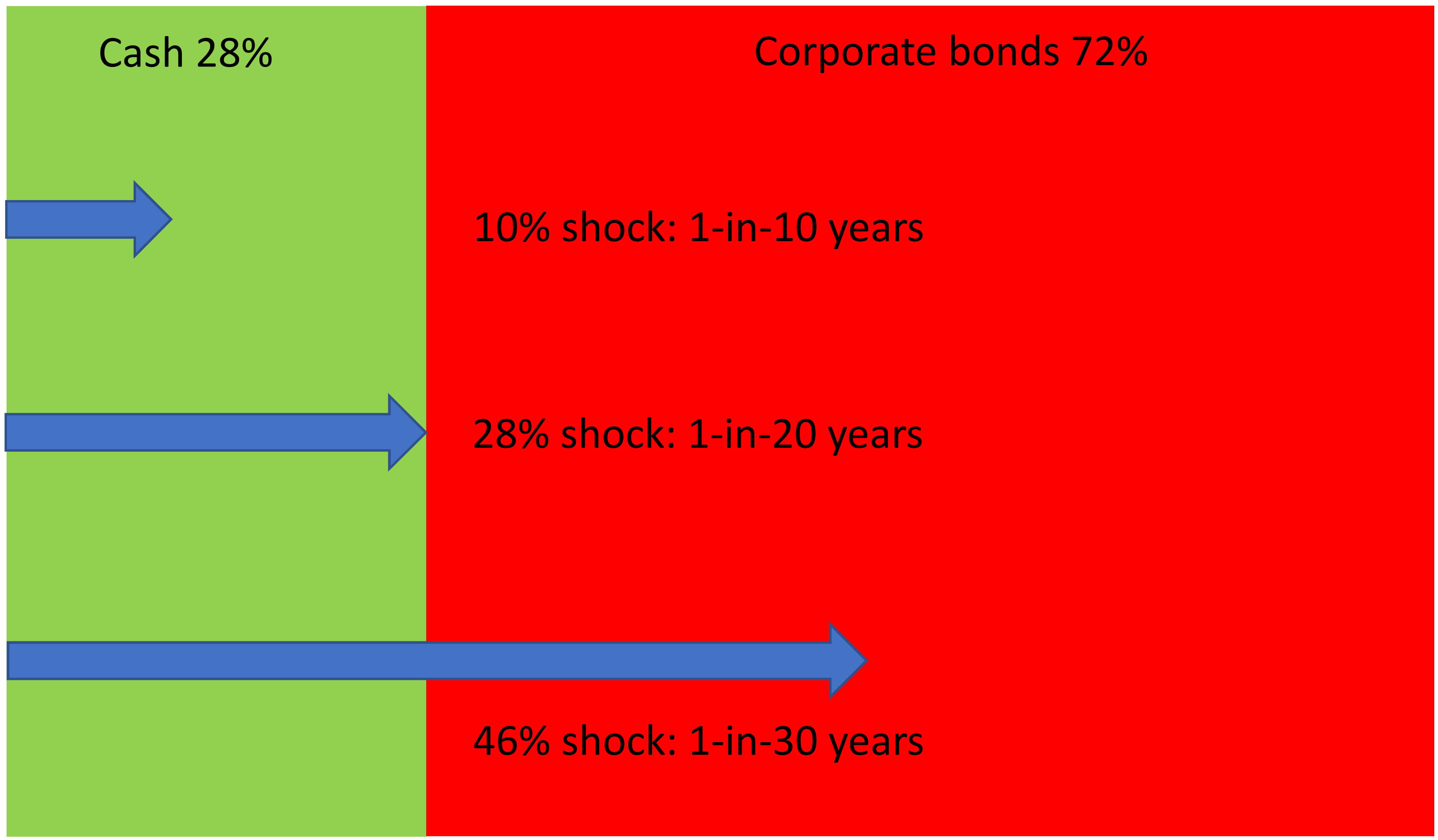
Optimisation: Life Fund

- Corporate bonds earn an illiquidity premium of 0.5% compared to a forced sale haircut of 10%.
- At the margin, the life fund can afford a forced sale of corporate bonds once every 20 years (=10%/0.5%). So we need enough cash to absorb a one-in-20 year liquidity shock.

Shock *s*	**1%**	**10%**	**28%**
Return period *r*(*s*)	5 years	10 years	20 years

- Assuming return periods are linear in the shock size, a shock of 28% occurs once every 20 years
- So with 28% in cash, and 72% in corporate bonds we are indifferent to marginal re-allocations between cash and bonds.
- There is positive net illiquidity premium because only the marginal bond is realised every 20 years. The other bonds are realised less frequently.

Life Fund Shock Example



Annuity Fund Example

- The calculation is similar to the life fund, but now the private placements also become attractive.
- Optimal portfolio in our example comes at:
    - Cash 1%
    - Corporate bonds 30%
    - Private bonds 69%
- Why this works: consider boundary between corporate and private bonds. This relates to a 31% liquidity shock, which (extrapolating linearly) occurs every 120 years.
- The marginal net IP's on both corporate and private bonds are both 0.42% if realised every 120 years, which is the first order optimality condition.

Illustrative Return Comparison

Fund	Gross IP p.a.	Haircut p.a.	Net IP p.a.
Hedge	0.00%	0.00%	0.00%
Life	0.36%	0.20%	0.16%
Annuity	0.67%	0.18%	0.49%

We exclude any profits that might be earned from other sources, such as risk premiums for market risk, default risk or expense risks. The illiquidity premium earned is a weighted average calculation, while the expected haircut involves mathematical integration with respect to distributions of shock sizes.
Linearity of $r(s)$ is for illustration only; empirical support needed.
We may have overstated the net IP if regulatory or other constraints prevent our preferred (most liquid first) sacrifice order.

Endogenous Illiquidity Models

- The operational risk approach fails to explain the haircuts; these are taken as exogenous inputs.
- It leaves a logical gap: who is benefiting from these haircuts, and why can insurers not be on the winning side?
- In economic theory, illiquidity often explained by information asymmetries (Kim & Verrecchia, 1994) to compensate dealers for adverse selection.
- Asymmetric information models can get complicated (because every participant has their own probabilities) and are not (yet) widely used.

Conclusions

- Much analysis in ALM concerns market + credit risk and expected returns. Illiquidity risk and return have been secondary considerations.
- We have shown a simple portfolio optimisation approach where illiquidity is the only risk.
- There is no fundamental theory of asset pricing or risk neutral measure if assets are illiquid. Future portfolio values after costs are non-linear in the constituents and the value of a security depends on more than its cash flows. This is a complex, poorly understood, world.
- Acknowledging the existence of an illiquidity premium does not justify adding it to a liability discount rate (especially if illiquidity costs are ignored)
- We need endogenous illiquidity models to explain who benefits from illiquidity costs.

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